MULTIDIMENSIONAL COSMOLOGY FOR INTERSECTING P-BRANES WITH STATIC INTERNAL SPACES

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Multidimensional cosmological model with static internal spaces describing the evolution of an Einstein space of non-zero curvature and n internal spaces is considered. The action contains several dilatonic scalar fields φ^I and antisymmetric forms A^I and Λ-term. When forms are chosen to be proportional to volume forms of p-brane submanifolds of internal space manifold, the Toda-like Lagrange representation arises. Exact solutions for the model are obtained, when scale factors of internal spaces are constant. It is shown that they are de Sitter or anti-de Sitter. Behaviour of cosmological constant and its generation by p-branes is demonstrated.

1 Introduction

Here we consider a multidimensional gravitational model governed by the action containing several dilatonic scalar fields and antisymmetric forms [4, 5, 7] and Λ-term with the aim of studying the solutions with static internal spaces. They are important if we want to have effective gravitational constant be really constant.

We study a cosmological sector of the model from [15] (see also [6]). We recall that the model from [15] incorporates generalized non-composite intersecting p-brane solutions. Using the σ-model representation from [15, 6] we reduce equations of motion to the pseudo-Euclidean Toda-like Lagrange system [11]–[14] with the zero-energy constraint.

Here we consider the case of constant vectors [16, 17, 18] in the Toda potential and obtain exact solutions for the system with p-branes. In this case we deal with the set of algebraic equations and only one differential equation for M_0 space. We investagate two possible cases, when all p-branes "live" on external manifold, and when p-brane world-sheet do not cover M_0 space.

In the first case using the synchronous-time parametrization we get the general exact solution for the initial cosmological metric. To get this solution we also use the so-called "fine-tuning" of the cosmological constant and curvatures of internal spaces. The second case turns out to be off interest due to independence of the solution on any p-brane

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configurations. In this case we have an ordinary multidimensional cosmological solution with Λ-term [16, 17]. We note that the solution with de Sitter and anti-de Sitter spaces are of interest now due to recent papers on M-theory on AdS/Superconformal theory duality [1, 2, 3].

2   The model

Here like in [15] we consider the model governed by the action

$$S = \frac{1}{2\kappa^2} \int_M d^D z \sqrt{|g|} \Big\{ R[g] - 2\Lambda - \sum_{I\in\Omega} \Big[g^{MN} \partial_M \varphi^I \partial_N \varphi^I + \frac{1}{n_I!} \exp(2\lambda_{JI}\varphi^J)(F^I)_g^2 \Big] \Big\},$$
(2.1)

where $g = g_{MN} dz^M \otimes dz^N$ is a metric, φ^I is a dilatonic scalar field,

$$F^I = dA^I = \frac{1}{n_I!} F^I_{M_1 \ldots M_{n_I}} dz^{M_1} \wedge \ldots \wedge dz^{M_{n_I}}$$
(2.2)

is a n_I-form ($n_I \geq 2$) on D-dimensional manifold M, Λ is the cosmological constant and $\lambda_{JI} \in \mathbf{R}$, $I, J \in \Omega$. In (2.1) we denote $|g| = |\det(g_{MN})|$,

$$(F^I)_g^2 = F^I_{M_1 \ldots M_{n_I}} F^I_{N_1 \ldots N_{n_I}} g^{M_1 N_1} \ldots g^{M_{n_I} N_{n_I}},$$
(2.3)

$I \in \Omega$, Here Ω is a non-empty finite set.

Equations of motion corresponding to (2.1) have the following form

$$R_{MN} - \frac{1}{2} g_{MN} R = T_{MN} - \Lambda g_{MN},$$
$$T_{MN} = \sum_{I\in\Omega} \Big[T^{\varphi^I}_{MN} + \exp(2\lambda_{JI}\varphi^J) T^{F^I}_{MN} \Big],$$
(2.4)

$$\triangle[g]\varphi^J - \sum_{I\in\Omega} \frac{\lambda_{JI}}{n_I!} \exp\Big(2\lambda_{KI}\varphi^K \Big) (F^I)_g^2 = 0,$$

$$\nabla_{M_1}[g] \Big(\exp(2\lambda_{KI}\varphi^K) F^{I, M_1 \ldots M_{n_I}} \Big) = 0,$$
(2.5)

$I, J \in \Omega$ and

$$T^{\varphi^I}_{MN} = \partial_M \varphi^I \partial_N \varphi^I - \frac{1}{2} g_{MN} \partial_P \varphi^I \partial^P \varphi^I,$$
(2.6)

$$T^{F^I}_{MN} = \frac{1}{n_I!} \Big[-\frac{1}{2} g_{MN} (F^I)_g^2 + n_I F^I_{MM_2 \ldots M_{n_I}} F_N^{I, M_2 \ldots M_{n_I}} \Big].$$
(2.7)

In (2.5) $\triangle[g]$ and $\nabla[g]$ are the Laplace-Beltrami and covariant derivative operators respectively corresponding to g.

We consider the manifold

$$M = \mathbf{R} \times M_0 \times \ldots \times M_n,$$
(2.8)

with the metric

$$g = w \, \mathrm{e}^{2\gamma(u)} \, du \otimes du + \sum_{i=0}^{n} \mathrm{e}^{2\phi^i(u)} \, g^i, \tag{2.9}$$

where $w = \pm 1$, u is a time variable and $g^i = g_{m_i n_i}(y_i) dy_i^{m_i} \otimes dy_i^{n_i}$ is a metric on M_i satisfying the equation

$$R_{m_i n_i}[g^i] = \xi_i g^i_{m_i n_i}, \tag{2.10}$$

$m_i, n_i = 1, \ldots, d_i$; $\xi_i = \mathrm{const}$, $i = 0, \ldots, n$. The functions $\gamma, \phi^i : \mathbf{R}_{\bullet} \to \mathbf{R}$ ($\mathbf{R}_{\bullet}$ is an open subset of $\mathbf{R}$) are smooth.

We consider any manifold M_i to be oriented and connected, $i = 0, \ldots, n$. Then, the volume d_i-form

$$\tau_i = \sqrt{|g^i(y_i)|} \, dy_i^1 \wedge \ldots \wedge dy_i^{d_i}, \tag{2.11}$$

and signature parameter $\varepsilon(i) = \mathrm{sign}(\det(g^i_{m_i n_i})) = \pm 1$ are correctly defined for all $i = 0, \ldots, n$.

Let Ω from (2.1) be a set of all non-empty subsets of $\{0, \ldots, n\}$. The number of elements in Ω is $|\Omega| = 2^{n+1} - 1$.

For any $I = \{i_1, \ldots, i_k\} \in \Omega$, $i_1 < \ldots < i_k$, we put in (2.2)

$$A^I = \Phi^I(u)\tau_{i_1} \wedge \ldots \wedge \tau_{i_k}, \tag{2.12}$$

where functions $\Phi^I : \mathbf{R}_{\bullet} \to \mathbf{R}$ are smooth, $I \in \Omega$, and τ_i are defined in (2.11). We denote by

$$d(I) \equiv d_{i_1} + \ldots + d_{i_k} = \sum_{i \in I} d_i \tag{2.13}$$

the dimension of the oriented manifold $M_I = M_{i_1} \times \ldots \times M_{i_k}$. It follows from (2.12) that

$$F^I = dA^I = d\Phi^I \wedge \tau_{i_1} \wedge \ldots \wedge \tau_{i_k}, \text{ and } n_I = d(I) + 1, \tag{2.14}$$

$I \in \Omega$.

Thus, dimensions of forms F^I in the considered model are fixed by a subsequent decomposition of the manifold.

For dilatonic scalar fields we put $\varphi^I = \varphi^I(u)$, $I \in \Omega$. Let

$$f = \gamma_0 - \gamma, \quad \sum_{i=0}^{n} d_i \phi^i \equiv \gamma_0. \tag{2.15}$$

It is not difficult to verify that the field equations (2.4)–(2.5) for field configurations from (2.9), (2.14) may be obtained as equations of motion corresponding to the action

$$S_\sigma = \frac{1}{2\kappa_0^2} \int du \, \mathrm{e}^f \Big\{ -w G_{ij} \dot{\phi}^i \dot{\phi}^j - w \delta_{IJ} \dot{\varphi}^I \dot{\varphi}^J -$$

$$w \sum_{I \in \Omega} \varepsilon(I) \exp\Big(2\vec{\lambda}_I \vec{\varphi} - 2\sum_{i \in I} d_i \phi^i\Big)(\dot{\Phi}^I)^2 - 2V \, \mathrm{e}^{-2f} \Big\}, \tag{2.16}$$

where $\vec{\varphi} = (\varphi^I)$, $\vec{\lambda}_I = (\lambda_{JI})$, $\dot{\varphi} \equiv d\varphi(u)/du$; $G_{ij} = d_i \delta_{ij} - d_i d_j$ are components of "pure cosmological" minisuperspace metric and

$$V = V(\phi) = \Lambda \, \mathrm{e}^{2\gamma_0(\phi)} - \frac{1}{2} \sum_{i=1}^{n} \xi_i d_i \, \mathrm{e}^{-2\phi^i + 2\gamma_0(\phi)} \tag{2.17}$$

is the potential. In (2.16) $\varepsilon(I) \equiv \varepsilon(i_1) \times \ldots \times \varepsilon(i_k) = \pm 1$ for $I = \{i_1, \ldots, i_k\} \in \Omega$.

3   Classical exact solutions

We consider the harmonic time gauge $\gamma = \gamma_0(\phi)$. The problem of integrability of the Lagrange equations arising in this model may be simplified if we integrate the Maxwell equations

$$\frac{d}{du}\left(\exp\left(2\vec{\lambda}_I\vec{\varphi} - 2\sum_{i\in I}d_i\phi^i\right)\dot{\Phi}^I\right) = 0, \quad \dot{\Phi}^I = Q^I\exp\left(-2\vec{\lambda}_I\vec{\varphi} + 2\sum_{i\in I}d_i\phi^i\right), \quad (3.1)$$

where Q^I are constant, $I \in \Omega$.

Let $Q^I \neq 0 \Leftrightarrow I \in \Omega_*$, where $\Omega_* \subset \Omega$ is some non-empty subset of Ω. For fixed $Q = (Q^I, I \in \Omega_*)$ the Lagrange equations corresponding to ϕ^i and φ^I, when equations (3.1) are substituted, are equivalent to Lagrange equations for the Lagrangian

$$L_Q = \frac{1}{2}\left[G_{ij}\dot{\phi}^i\dot{\phi}^j + \delta_{IJ}\dot{\varphi}^I\dot{\varphi}^J\right] - V_Q \qquad (3.2)$$

where

$$V_Q = V_Q(\phi, \varphi, w) = -wV(\phi) + \frac{1}{2}\sum_{I\in\Omega_*}\varepsilon(I)\exp\left(-2\vec{\lambda}_I\vec{\varphi} + 2\sum_{i\in I}d_i\phi^i\right)(Q^I)^2. \qquad (3.3)$$

($V(\phi)$ is defined in (2.17)). Thus, we are led to the pseudo-Euclidean Toda-like system (see [9], [11]–[14]) with the zero-energy constraint:

$$L_Q = \frac{1}{2}\bar{G}_{AB}\dot{x}^A\dot{x}^B - V_Q, \quad E_Q = \frac{1}{2}\bar{G}_{AB}\dot{x}^A\dot{x}^B + V_Q = 0, \qquad (3.4)$$

where $x = (x^A) = (\phi^i, \varphi^I)$,

$$(\bar{G}_{AB}) = \begin{pmatrix} G_{ij} & 0 \\ 0 & \delta_{IJ} \end{pmatrix} \qquad (3.5)$$

$i, j \in \{0, \ldots, n\}$, $I, J \in \Omega$, and the potential V_Q may be presented in the following form

$$V_Q = \sum_{i=0}^{n}\left(\frac{w}{2}\xi_i d_i\right)\exp[2U^i(x)] - (w)\Lambda\exp[2U^\Lambda(x)]$$

$$+ \sum_{I\in\Omega_*}\frac{\varepsilon(I)}{2}(Q^I)^2\exp[2U^I(x)], \qquad (3.6)$$

where

$$U^\Lambda(x) = U_A^\Lambda x^A = \sum_{j=0}^{n}d_j\phi^j, \quad U^i(x) = -\phi^i + \sum_{j=0}^{n}d_j\phi^j,$$

$$U^I(x) = \sum_{i\in I}d_i\phi^i - \vec{\lambda}_I\vec{\varphi}, \qquad (3.7)$$

$i, j \in \{0, \ldots, n\}$, $I \in \Omega_*$, $J \in \Omega$. The contravariant components of these vector read

$$U^{Ii} = G^{ij}U_j^I = \delta_i^I - \frac{d(I)}{D-2}, \quad U^{\Lambda i} = \frac{1}{2-D}, \quad U^{ji} = \frac{\delta^{ji}}{d_i}, \qquad (3.8)$$

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where

$$\delta_i^I \equiv \sum_{j \in I} \delta_{ij} = \begin{cases} 1, & i \in I \\ 0, & i \notin I \end{cases}, \quad G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2-D}, \tag{3.9}$$

are the indicator of belonging i to I and the matrix inverse to the matrix (G_{ij}) correspondingly, $i, j = 0, \ldots, n$.

Now let us continue with the case, when the scale factors of the internal spaces are constant.

4 The case of constant internal space factors

The Lagrange system (3.4) leads to the following equations of motion

$$\bar{G}_{AB}\ddot{x}^B + \frac{\partial V_Q}{\partial x^A} = 0. \tag{4.1}$$

Substituting the potential (3.6) into (4.1) we get the set of equations

$$\ddot{\phi}^j - (w\xi_j)\exp[2U^j(x)] - \frac{2w}{2-D}\Lambda\exp[2U^\Lambda(x)]$$

$$+ \sum_{I \in \Omega} \varepsilon(I)(Q^I)^2 \left(\delta_j^I - \frac{d(I)}{D-2}\right)\exp[2U^I(x)] = 0, \tag{4.2}$$

$$\ddot{\varphi}^J - \sum_{I \in \Omega} \varepsilon(I)(Q^I)^2 \lambda_I^J \exp[2U^I(x)] = 0. \tag{4.3}$$

Now let us consider the special case with the $x^k = x_0^k = \text{const}$, $k = 1, \ldots, n$; $\varphi^I = \text{const}$, $I \in \Omega$. This case corresponds to the case with static internal spaces, when sizes of internal dimensions may be set arbitrary small and so unobservable during the whole evolution of the Universe. Then the equations of motion read

$$\ddot{\phi}^0 - \xi_0 A_0 \exp[(2d_0 - 2)\phi^0] - \Lambda A_\Lambda \exp[(2d_0)\phi^0] + \sum_{I \in \Omega} A_I^0 \exp[(2d_0\delta_0^I)\phi^0] = 0, \tag{4.4}$$

$$- \xi_k A_k \exp[(2d_0)\phi^0] - \Lambda A_\Lambda \exp[(2d_0)\phi^0] + \sum_{I \in \Omega} A_I^k \exp[(2d_0\delta_0^I)\phi^0] = 0, \tag{4.5}$$

$$\sum_{I \in \Omega} \lambda_I^J A_I^0 \left(\delta_0^I - \frac{d(I)}{D-2}\right)^{-1} \exp[(2d_0\delta_0^I)\phi^0] = 0. \tag{4.6}$$

where $k = 1, \ldots, n$, $J \in \Omega$ and

$$A_0 = w\prod_{l=1}^n (X^l)^{d_l}, \quad A_k = \frac{w}{X^k}\prod_{l=1}^n (X^l)^{d_l}, \quad A_\Lambda = \frac{2w}{2-D}\prod_{l=1}^n (X^l)^{d_l}, \tag{4.7}$$

$$A_I^k = \varepsilon(I)(\tilde{Q}^I)^2 \left(\delta_k^I - \frac{d(I)}{D-2}\right)\prod_{l \in I\setminus\{0\}} (X^l)^{d_l}. \tag{4.8}$$

where we introduce new variables X^k: $X^k \equiv \exp[2\phi^k]$, $i = 0, \ldots, n$, $k = 1, \ldots, n$ and $\tilde{Q}^I = Q^I \exp[-\vec{\lambda}_I\vec{\varphi}]$. Thus we have one differential equation (4.4) for ϕ^0, the set of algebraic equations (4.5) for ϕ^k and equations (4.6) for φ^I, $I \in \Omega$.

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As we can see, there are two possibilities to solve equations (4.4), (4.5): $0 \in I$, $\forall I \in \Omega$ or $0 \notin I$, $\forall I \in \Omega$, i.e. when forms are present in our external space or they are completely defined on internal subspaces.

4.1 The case with all p-branes "living" also in external space

Let us consider the first case. The equation (4.4) takes the form

$$\ddot{\phi}^0 - \xi_0 A_0 \exp[(2d_0 - 2)\phi^0] + \{-\Lambda A_\Lambda + A_\Omega^0\} \exp[(2d_0)\phi^0] = 0, \quad A_\Omega^0 = \sum_{I \in \Omega} A_I^0. \quad (4.9)$$

The first integral of the equation (4.9) reads

$$\frac{(\dot{\phi}^0)^2}{2} - \frac{\xi_0 A_0}{2d_0 - 2} \exp[(2d_0 - 2)\phi^0] - \frac{\Lambda A_\Lambda - A_\Omega^0}{2d_0} \exp[2d_0\phi^0] = E', \quad (4.10)$$

where due to the zero-energy constraint (3.4) E' takes the form

$$-E' = \sum_{k=1}^{n} \frac{\xi_k d_k}{2d_0(1 - d_0)} A_k \exp[(2d_0)\phi^0] + \frac{\Lambda A_\Lambda}{2d_0} \left(\frac{D-2}{1-d_0} + 1\right) \exp[(2d_0)\phi^0]$$

$$+ \frac{\exp[(2d_0)\phi^0]}{2d_0} \sum_{I \in \Omega} A_I^0 \left[\frac{1}{1 - d_0}\left(1 - \frac{d(I)}{D-2}\right)^{-1} - 1\right], \quad (4.11)$$

but if one sums the equations (4.5) with the weights d_k, one can get that $E' = 0$. Thus, we have the zero-energy constraint for the subsystem with ϕ^0 that is usefull for the models with inflation. Finally we get the following solution of (4.9)

$$\int^{\phi^0} \frac{d\bar{\phi}^0}{\sqrt{\frac{\xi_0 A_0}{d_0 - 1} \exp[(2d_0 - 2)\bar{\phi}^0] + \frac{\Lambda A_\Lambda - A_\Omega^0}{d_0} \exp[2d_0\bar{\phi}^0]}} = u - u_0, \quad (4.12)$$

where $u_0 = \text{const}$.

The equation (4.5) in this case takes the following form

$$\left(-\frac{w\xi_k}{X^k} - \frac{2\Lambda w}{2 - D}\right) \prod_{l=1}^{n} (X^l)^{d_l} + \sum_{I \in \Omega} \varepsilon(I)(\tilde{Q}^I)^2 \left(\delta_k^I - \frac{d(I)}{D-2}\right) \prod_{l \in I \setminus \{0\}} (X^l)^{d_l} = 0, \quad (4.13)$$

$k = 1, \ldots, n$.

4.2 The case with all p-branes not "living" in our space

In the second case $0 \notin I$, $\forall I \in \Omega$ to integrate equations (4.5) we must put the following restriction on the scale factors of internal spaces

$$X^k = \xi_k \frac{D-2}{2\Lambda}. \quad (4.14)$$

This restriction let us cancel the terms with the scale factor of the M_0 space. Such a method called "fine-tuning" was previously used in [16] to obtain cosmological solutions with static internal spaces, but without p-branes. So, we get simple equations for the internal space factors:

$$\sum_{I \in \Omega} \varepsilon(I)(\tilde{Q}^I)^2 \left(\delta_k^I - \frac{d(I)}{D-2} \right) \left(\frac{D-2}{2\Lambda} \right)^{d(I)} \prod_{l \in I} (\xi_l)^{d_l} = 0. \tag{4.15}$$

$k = 1, \ldots, n$. Equation (4.4) in this case takes the form

$$\ddot{\phi}^0 - \xi_0 A_0 \exp[(2d_0 - 2)\phi^0] - \Lambda A_\Lambda \exp[(2d_0)\phi^0] + A_\Omega^0 = 0. \tag{4.16}$$

But if we sum the equations (4.15) with the weights d_k we get that $A_\Omega^0 = 0$ for this case. We also have the zero-energy constraint for the subsystem with ϕ^0 in this case. Thus the solution reads

$$\int^{\phi^0} \frac{d\bar{\phi}^0}{\sqrt{\dfrac{\xi_0 A_0}{d_0 - 1} \exp[(2d_0 - 2)\bar{\phi}^0] + \dfrac{\Lambda A_\Lambda}{d_0} \exp[2d_0\bar{\phi}^0]}} = u - u_0. \tag{4.17}$$

As we can see this solution does not depend on any p-brane properties. So, this is the cosmological solution with Λ-term (see [16]). Thus, if p-branes do not "live" on M_0 and they are compensated in internal spaces (see (4.13)), we can not "feel" the influence of the branes configurations.

5 Some examples and conclusion

Now we consider some examples of the obtained solutions. As we can see, only the first case when $0 \in I$, $\forall I \in \Omega$ is interesting as p-branes solutions. So, let us investigate the solutions in this case. It is usefull to use the synchronous-time parametrization for the solutions. Let t_s: $dt_s = e^\gamma du$, be a synchronous time. Here we put $w = -1$. Then the solution (4.12) takes the following form

$$\int^{\phi^0} \frac{d\phi^0}{\sqrt{\dfrac{\xi_0}{1 - d_0} \exp[-2\phi^0] - \dfrac{\Lambda A_\Lambda - A_\Omega^0}{d_0 A_0}}} = t_s. \tag{5.1}$$

Let us consider some special case. Here we also use the "fine-tuning" of a cosmological constant Λ, and put

$$\xi_0 = d_0 - 1, \quad \xi_i = \frac{2\Lambda}{D - 2}, \tag{5.2}$$

$i = 1, \ldots, n$. Then from (5.1) we get

$$g = -dt_s \otimes dt_s + \frac{\mathrm{ch}^2[Ht_s]}{H^2} g^0 + \sum_{k=1}^n g^k, \tag{5.3}$$

where

$$d_0 H^2 = \frac{\Lambda A_\Lambda - A_\Omega^0}{-A_0} = \frac{2\Lambda}{D-2} - \sum_{I \in \Omega} \varepsilon(I)(\tilde{Q}^I)^2 \left(1 - \frac{d(I)}{D-2}\right) \equiv \bar{\xi}_0. \tag{5.4}$$

Thus, for the new metric

$$\bar{g}^0 \equiv -dt_s \otimes dt_s + \frac{\mathrm{ch}^2[Ht_s]}{H^2} g^0, \tag{5.5}$$

we have $\mathrm{Ric}[\bar{g}^0] = \bar{\xi}_0 \bar{g}^0$. As we can see, for $(M_0, g^0) = (S^{d_0}, g[S^{d_0}] = d\Omega_{d_0}^2)$ we get the de Sitter space. In this case the relations (4.6) and (4.13) read

$$\sum_{I \in \Omega} \varepsilon(I)(\tilde{Q}^I)^2 \left(\delta_k^I - \frac{d(I)}{D-2}\right) = 0, \quad \sum_{I \in \Omega} \lambda_I^J \varepsilon(I)(\tilde{Q}^I)^2 = 0, \tag{5.6}$$

$k = 1, \ldots, n$, $J \in \Omega$. For the form F^I we also get

$$F^I = \tilde{Q}^I \, \mathrm{e}^{-\vec{\lambda}_I \vec{\varphi}} \, \bar{\tau}_0 \wedge \tau(\bar{I}), \tag{5.7}$$

where $\bar{\tau}_0$ is the volume form on dS^{d_0+1}, $\bar{I} = I \setminus \{0\}$, $I \in \Omega$.

Example. Now let us consider some more special cases. Here we put $n = 2$ and

$$\Omega = \{\{0\}, \{0, 1\}, \{0, 2\}\}. \tag{5.8}$$

Thus, resolving the first equation in (5.6) we get the following formulas for the charges of forms

$$(\tilde{Q}^0)^2 = -\frac{\varepsilon(2)(d_0 + 1)}{d_0} \tilde{Q}^2, \quad (\tilde{Q}^{0,1})^2 = \frac{\varepsilon(2)}{\varepsilon(1)} \tilde{Q}^2, \quad (\tilde{Q}^{0,2})^2 = \tilde{Q}^2, \tag{5.9}$$

where $\tilde{Q}^2$ is some arbitrary constant. Here, as we can see, one must put $\varepsilon(1) = \varepsilon(2) = -1$. Thus, the effective cosmological term of the $\bar{g}^0$ space in this case reads

$$\frac{2\tilde{\Lambda}}{d_0 - 1} = \frac{2\Lambda}{D-2} - \frac{\varepsilon(0)}{d_0} \tilde{Q}^2, \tag{5.10}$$

so, for the case with $(M_0, g^0) = (S^{d_0}, g[S^{d_0}] = d\Omega_{d_0}^2)$ ($\bar{g}^0$ is de Sitter space) $\varepsilon(0) = +1$ and we may get the extremely small effective cosmological constant $\tilde{\Lambda}$ while the multidimensional cosmological bare constant Λ has the Planck scale.

The case with $\Lambda = 0$. If we put $\Lambda = 0$, then due to the equation (4.13) we come to the case with Ricci-flat internal spaces, i.e. $\xi_k = 0$, $k = 1, \ldots, n$, without any other changes in the results of the section 5. But inflationary solutions are generated in this case only due to the fields of forms (p-branes).

Thus, we considered the multidimensional cosmology, with $n + 1$ Einstein spaces of non-zero curvature (M_i, g^i), $i = 0, \ldots, n$ in the presence of several scalar fields and forms. When scale factors of the internal spaces (M_k, g^k), $k = 1, \ldots, n$, are chosen to be constant, we obtained the exact solution, describing the evolution of one external

space (M_0, g^0) in the presence of n "frozen" internal spaces. There were investigated two possible cases: $0 \in I$, $\forall I \in \Omega$ and $0 \notin I$, $\forall I \in \Omega$. The second case turned out to be off interest because p-brane configurations do not have any influence on a scale factor of the external space. In this case we had an ordinary cosmological solution with Λ-term for M_0.

In the other case the influence of p-branes is rather important. There we got the solution for the metric of M_0 in the synchronous-time parametrization, that allowed us to investigate this solution in detail. Although we considered the de Sitter type solution, we may also obtain the anti-de Sitter one for another sign of curvature. So, the solution may be de Sitter or anti-de Sitter one depending on p-brane configurations. In this case it is also necessary to use "fine-tuning" of the cosmological constant and curvatures of internal spaces to obtain the solutions. This allows us to make the effective cosmological constant $\tilde{\Lambda}$ extremely small while the multidimensional cosmological constant Λ has the Planck scale.

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